PROPERTIES OF INDEFINITE INTEGRALS, METHODS OF ITS CALCULATION AND, ACCORDINGLY, INTEGRATION OF SOME TRIGONOMETRIC FUNCTIONS

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***Abstract**: in this scientific article, we focused on the integral section, which is one of the important sections of "Higher Mathematics". Here we will consider the inverse of Differentiation by Integrating. The article shows that the act of differentiation consists in finding the derivative of a function defined in an area E in this area or at a point x0 of the area E. Therefore, Integrating consists of finding a function based on a given derivative, which includes the integral, which is the main concept of differential equations, and the sample examples for each part are solved in a sufficiently simple and understandable way.*

Key word: *initial function, properties of indefinite integral, table of integrals, methods of calculating indefinite integral, integration of rational fractions, integration of some trigonometric functions.*

Initial function. Let us be given the function *y=f(x)*. The act of finding the derivative of this function is called differentiation of the function. For example, if the motion *s=f(t)* is given, we can find the velocity *"v"* by differentiating it with respect to *"t"*. If we differentiate this speed by *"t"*, we will find the acceleration.

However, in practice, it is also necessary to solve the opposite problem: that is, the acceleration $a=a(t)$ is given as a function of time t, and it is requested to determine the distance *"s"* and speed *"v"* traveled at time *"t"*.

So, here it is necessary to find the function $v=v(t)$ with the derivative $a=a(t)$, and then find the function $s=s(t)$ with the derivative v.

In many problems, it is necessary to find the derivative of an unknown function. If the function $f(x)$ is given, it is necessary to find such a function $F(x)$ that its derivative is equal to the given function, that is:

$$F'(x)=f(x)$$

Description. If the equality $F'(x)=f(x)$ holds at each point of the section $[a,b]$, then the function $F(x)$ is called the initial function of the given function $f(x)$.

For example, let $f(x)=x^2$ be given. Its initial function will be $F(x)=x^4/4$, because:

$F'(x)=(x^4/4)'=4x^3/4=x^3$

Example 1. If $f(x)=$ then its initial function is equal to $F(x)=tgx$. Because:

$$F'(x)=(tgx)'=\frac{1}{cos^2 x}$$

If a function $f(x)$ has an initial function $F(x)$, then any other initial function of $f(x)$ is invariantly different from $F(x)$.

For example, let $F(x)$ be the initial function of the given function $f(x)$.

Let $f(x)$ be another initial function of $f(x)$; where $F(x)=F(x)+C$ where *"C"* is a constant quantity. The following conclusion follows from this.

If $F(x)$; If $f(x)$ is an initial function, then $F(x)+C$ is also an initial function of $f(x)$, which is the set of all initial functions of $f(x)$.

It follows that there are infinitely many initial functions of the function $f(x)$.

For example: $f(x)=x^3$; $F(x)=x^4/4$ was. But $F(x)=x^4/4+C$ is also an initial function. Because $F'(x)=(x^4/4+C)'=x^3$

Now we give the definition of indefinite integral. Definition: If the function $F(x)$ is an initial function of the function $f(x)$, then the expression $F(x)+C$ is also an initial function and is called the indefinite integral of the function $f(x)$ and $\int f(x)dx$ is determined in appearance.

Here, $f(x)$ is a function under integral, □ symbol is called an integral symbol. Thus, the indefinite integral $y=F(x)+C$ consists of a set of functions.

The geometric meaning of an indefinite integral consists of a family of lines in a plane, that is, straight lines or curves, which are parallel to the line itself, along the *"X"* or *"Y"* axis., will consist of moving down or up. It changes depending on whether the argument accepts negative or positive values.

Any continuous function has an initialization function. So there is an indefinite integral of such a function.

Integrating a function means finding its initial function. Therefore, when integrating a function, the result of integration is checked by deriving from the initial function found.

Properties of the indefinite integral.

1. d(□f(x)dx)=f(x)dx

2. $\int$ df(x)=f(x)+C (C-const)

3. $\int$ f(ax+b)dx=1/aF(ax+b)+C (a,b-const)

4. ($\int$ f(x)dx)′=(F(x)+C)′=f(x).

5. The indefinite integral of the algebraic sum of several functions is equal to the algebraic sum of the integrals of these functions, i.e.:

$\int [f_1(x)+f_2(x)+...+f_n(x)]dx=\int f_1(x)dx+\int f_2(x)dx+...+\int f_n(x)dx$

6. The constant multiplier can be taken out of the integral sign. that is, if, then:

$\int af(x)dx=a\int f(x)dx$ will be.

These properties can be easily proved using the integral definition. Proof of this will be given to the students.

Table of integrals. Now we present the table of integrals. The table of integrals directly follows from the table of derivatives. The correctness of the equations given in the table can be checked by differentiation, that is, it is possible to determine whether the derivative of the function on the right side of the equation is equal to the function under the integral.

1. $\int x^n dx=\frac{x^{n+1}}{n+1}+C$ $(C=const,\ n\neq -1)$;	8. $\int \frac{dx}{\sin^2 x}=-ctgx+C$ $(C=const)$;
2. $\int \frac{1}{x}dx=\ln\|x\|+C$ $(C=const)$;	9. $\int tgxdx=-ln\|cosx\|+C$ $(C=const)$;
3. $\int a^x dx=\frac{a^x}{\ln a}+C$ $(C=const,\ a>0)$;	10. $\int ctgxdx=ln\|sinx\|+C$ $(C=const)$;
4. $\int e^x dx=e^x+C$ $(C=const)$;	11. $\int lnxdx=xlnx-x+C$ $(C=const)$;
5. $\int sinxdx=-cosx+C$ $(C=const)$;	12. $\int xdx=x^2/2+C$ $(C=const)$;
6. $\int cosxdx=sinx+C$ $(C=const)$;	13. $\int dx=x+C$ $(C=const)$;
7. $\int \frac{dx}{\cos^2 x}=tgx+C$ $(C=const)$;	

14. $\int \frac{dx}{x^2+a^2}=\frac{1}{a}arctg\frac{x}{a}+C=-\frac{1}{a}arctg\frac{x}{a}+C.$ $(C=const, a\neq 0)$;

15. $\int \frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C$ $(C=const, a\neq 0)$;

16. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln|\frac{x+a}{x-a}| + C \quad (C = const, a \neq 0)$;

17. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x + \sqrt{x^2 \pm a^2}| + C \quad (C = const, a \neq 0)$;

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C = -\arccos\frac{x}{a} + C \quad (C = const, a > 0)$;

19. $\int \frac{dx}{\sin x} = \ln|tg\frac{x}{2}| + C = \ln|\operatorname{cosec} x - ctgx| + C \quad (C = const)$;

20. $\int \frac{dx}{\cos x} = \ln|tg(\frac{x}{2} + \frac{\pi}{4})| + C = \ln|tgx + \sec x| + C \quad (C = const)$.

The indefinite integral can be calculated using the properties of the indefinite integral given above and the table of integrals.

For example: $\int \frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}}dx$ be given. We calculate this integral:

$$\int \frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}}dx = \int \frac{x^4 - 2x^2 + x^2 - 2}{x^{2/3}}dx = \int \frac{x^4 - x^2 - 2}{x^{2/3}}dx =$$

$$= \int (\frac{x^4}{x^{2/3}} - \frac{x^2}{x^{2/3}} - \frac{2}{x^{2/3}})dx = \int x^{10/3}dx - \int x^{4/3}dx - 2\int x^{-2/3}dx =$$

$$= \frac{x^{10/3+1}}{10/3+1} - \frac{x^{4/3+1}}{4/3+1} - 2\frac{x^{-2/3+1}}{-2/3+1} + C = \frac{3}{13}x^{13/3} - \frac{3}{7}x^{7/3} - 6x^{1/3} + C$$

Now, if we take the derivative from the found expression, and the function under the integral is derived, the found result will be correct. Indeed, $(\frac{3}{13}x^{3/13} - \frac{3}{7}x^{7/3} - 6x^{1/3} + C)' = x^{10/3} - x^{4/3} - 2x^{-2/3} =$

$$= \frac{x^4 - x^2 - 2}{x^{2/3}} = \frac{x^4 - 2x^2 + x^2 - 2}{\sqrt[3]{x^2}} = \frac{x^2(x^2-2) + (x^2-2)}{\sqrt[3]{x^2}} = \frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}}.$$

So the result is correct.

Methods of calculating the indefinite integral: When calculating the indefinite integral, the initial function of the function under the integral is found. This initial function is found using the properties of integrals given above and the table of integrals. In addition, variable substitution and piecewise integration methods are used in integration.

a) the method of replacing or substituting a variable. In integration with this method, the variable *"x"* is replaced by a new variable *"t"* in a certain relationship, so that the result is a simple integral.

To us $\int f(x)dx$ be given. $x=\varphi(t)$ let's take the permutation. From this $dx=\varphi^{/}(t)dt$ If we find and put it in the given integral, we get the following:

$$\int f(x)dx=\int f[\varphi(t)]\varphi^{/}(t)dt$$

This is much simpler than the given integral. When calculating the integral in general, the integral given by various substitutions is brought to one of the integrals in the table. Then the initial function is determined from the table.

Sometimes in the given integral $x=\varphi(t)$ instead of $t=\psi(x)$ replacement works well. If integral $\int\frac{\psi^{/}(x)}{\psi(x)}dx$ if it is given in the form, then $t=\psi(x)$ by substitution, the integral becomes very simple. Indeed, $t=\psi(x)$; $dt=\psi^{/}(x)dx$;

$$\int\frac{\psi^{/}(x)dx}{\psi(x)}=\int\frac{dt}{t}=\ln|t|+C=\ln|\psi(x)|+C.$$

It can be seen that when integrating a variable with replacement, the result is expressed again using the previous variable, i.e., it goes from the variable *"t"* to the variable *"x"*.

An example. Calculate the following integral: $\int\frac{\sin xdx}{\sqrt{1+2\cos x}}$ in which: *1+2cosx=t* we can say. In this case *-2sinxdx=dt* will be. So,

$$\int \frac{\sin x dx}{\sqrt{1+2\cos x}} = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2}\int t^{-1/2}dt = -\frac{1}{2}2t^{1/2} + C = -\sqrt{t} + C = -\sqrt{1+2\cos x} + C$$

b) Piecewise integration method. Let us be given two differentiable functions *"u(x)"* and *"v(x)"*. Let's find the differential of the product of these functions *"(uv)"*. This differential is defined as: *d(uv)=udv+vdu*

Integrating both sides of this step by step, we get the following:

$$uv = \int udv + \int vdu$$

or $\int udv = uv - \int vdu$ (1)

The last expression found is called the formula for integration by pieces. When calculating the integral using this formula $\int udv$ integral in appearance, is much simpler $\int vdu$ is brought to the integral of the form. If under the integral *"u=lnx"* function, or the product of two functions, and inverse trigonometric functions are involved, the formula for integration by pieces is used. When integrating with this method, there is no need to switch to a new variable.

In general, when calculating the indefinite integral, it is necessary to add a constant *(S=const)* next to the result. Otherwise, one value of the integral will be found and the rest will be discarded. This is considered an integration error.

Example: $\int xarctgxdx$ calculate the.

(where *S=0* is taken). We use formula (1).

$$\int xarctgxdx = \frac{x^2}{2}arctgx - \int \frac{x^2}{2(1+x^2)}dx \qquad (*)$$

$$\int \frac{x^2}{1+x^2}dx$$

we calculate separately:

$$\int \frac{x^2}{1+x^2}dx = \int \frac{1+x^2-1}{1+x^2}dx = \int (1-\frac{1}{1+x^2})dx = x - arctgx + C$$

we put this in (*).

$$\int xarctgxdx = \frac{x^2}{2}arctgx - \frac{x}{2} + \frac{1}{2}arctgx + C = -\frac{1}{2} + \frac{x^2+1}{2}arctgx + C$$

c*) Integrals of a function involving quadratic triplets*. Such integrals are mainly in the following form:

$$1. J_1 = \int \frac{dx}{ax^2+bx+c}; \quad 2. J_2 = \int \frac{Ax+B}{ax^2+bx+c}dx; \quad 3. J_3 = \int \frac{dx}{\sqrt{ax^2+bx+c}};$$

$$4. J_4 = \int \frac{Ax+B}{\sqrt{ax^2+bx+c}}dx; \quad 5. J_5 = \int \sqrt{ax^2+bx+c}dx.$$

To calculate such integrals, the complete square is separated from the triad under the integral, and it is brought to the algebraic sum of the square of the triad. The resulting expression can be integrated using the table of integrals. A complete square is separated from a square triangle as follows:

$$ax^2+bx+c = a(x^2+\frac{b}{a}x+\frac{c}{a}) = a[(x+\frac{b}{2a})^2+\frac{c}{a}-\frac{b^2}{4a^2}] = a[(x+\frac{b}{2a})^2 \pm k^2]$$

(in this place: $\pm k^2 = \frac{b^2-4ac}{4a^2}$)

In this case, the plus or minus sign is determined depending on whether the roots of the square triangle *"ax2+bx+c"* are real or complex, that is, it is determined depending on the sign of *"b2-4ac"*.

After dividing the complete square, the above integrals take the following form.

$$1. J_1 = \int \frac{dx}{ax^2+bx+c} = \frac{1}{a}\int \frac{dx}{(x+\frac{b}{2a})^2 \pm k^2}$$

In this *x+b/2a=t*, *dx=dt* let's say,

$$J_1 = \frac{1}{a}\int \frac{dt}{t^2 \pm k^2}$$

which is the integral in the table.

An example. $\int \frac{dx}{2x^2 + 8x + 20}$ be counted.

Solving. $\int \frac{dx}{2x^2 + 8x + 20} = \frac{1}{2}\int \frac{dx}{x^2 + 4x + 10} = \frac{1}{2}\int \frac{dx}{x^2 + 4x + 4 + 10 - 4} =$

$$= \frac{1}{2}\int \frac{dx}{(x+2)^2 + 6} = J; x + 2 = t dx = dt;$$

$$J = \frac{1}{2}\int \frac{dt}{t^2 + 6} = \frac{1}{2}\frac{1}{\sqrt{6}} arctg \frac{1}{\sqrt{6}} + C;$$

We find the final result by putting the expression through $''x''$ instead of $''t''$.

$$J = \frac{1}{2\sqrt{a}} arctg \frac{x+2}{\sqrt{6}} + C$$

$$2. J_2 = \int \frac{Ax + B}{ax^2 + bx + c} dx = \int \frac{\frac{A}{2a}(2ax + b) + (B - \frac{Ab}{2a})}{ax^2 + bx + c} dx =$$

$$= \frac{A}{2a}\int \frac{2ax + b}{ax^2 + bx + c} dx + (B - \frac{Ab}{2a})\int \frac{dx}{ax^2 + bx + c}$$

$$I = \int \frac{(2ax+b)dx}{ax^2 + bx + c} = \begin{bmatrix} ax^2 + bx + c = t \\ (2ax + b)dx = dt \end{bmatrix} = \int \frac{dt}{t} = \ln|t| + C = \ln|ax^2 + bx + c| + C$$

$$J_2 = \frac{A}{2a}\ln|ax^2 + bx + c| + (B - \frac{Ab}{2a})J_1$$

An example. $J = \int \frac{x+3}{x^2 - 2x - 5} dx$ be counted.

$$\frac{1}{2}\ln|x^2 - 2x - 5| + 4\int \frac{dx}{(x-1)^2 - 6} = \frac{1}{2}\ln|x^2 - 2x - 5| + 2\frac{1}{\sqrt{6}}\ln|\frac{\sqrt{6} - (x-1)}{\sqrt{6} + (x-1)} + C$$

As a result of the substitutions seen above, this integral is reduced to the following form:

when a>0; $J_3 = \int \frac{dt}{\sqrt{t^2 \pm k^2}}$;

when a<0; $J_3 = \int \frac{dt}{\sqrt{k^2 - t^2}}$.

And these are the integrals in the table.

An example. $\int \frac{dx}{\sqrt{x^2 - 4x - 3}}$ be counted.

x^2-4x-3=(x-2)^2-7; *dx=d(x-2)*

$$\int \frac{dx}{\sqrt{x^2 - 4x - 3}} = \int \frac{d(x-2)}{\sqrt{(x-2)^2 - 7}} = \ln\left|x - 2 + \sqrt{(x-2)^2 - 7}\right| + C$$

was calculated based on the integral in the table.

$$4.\, J_4 = \int \frac{Ax + B}{\sqrt{ax^2 + bx + c}} dx;$$

$$J_4 = \int \frac{Ax + B}{\sqrt{ax^2 + bx + c}} dx = \int \frac{\frac{A}{2a}(2ax + b) + (b - \frac{Ab}{2a})}{\sqrt{ax^2 + bx + c}} dx = \frac{A}{2a} \int \frac{2ax + b}{\sqrt{ax^2 + bx + c}} dx +$$

$$(B - \frac{Ab}{2a}) \int \frac{dx}{\sqrt{ax^2 + bx + c}}.$$

$$I = \int \frac{(2ax + b)dx}{\sqrt{ax^2 + bx + c}} = \begin{bmatrix} ax^2 + bx + c = t \\ (2ax + b)dx = dt \end{bmatrix} = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + C = 2\sqrt{ax^2 + bx + c} + C$$

$$J_4 = \frac{A}{a}\sqrt{ax^2 + bx + c} + (B - \frac{Ab}{2a}) J_3$$

An example. $\int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}} dx$ be counted.

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \int \frac{5/2(2x+4)+(3-10)}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}\int \frac{(2x+4)dx}{\sqrt{x^2+4x+10}} -$$

$$-7\int \frac{dx}{\sqrt{(x+2)^2+6}} = 5\sqrt{x^2+4x+10} - 7\ln\left|x+2+\sqrt{(x+2)^2+6}\right| + C$$

$$= 5\sqrt{x^2+4x+10} - 7\ln\left|x+2+\sqrt{x^2+4x+10}\right| + C$$

5. $J_5 = \int \sqrt{ax^2+bx+c}dx$ In this case, we divide the square triangle under the integral into a full square.

$$J_5 = \int \sqrt{ax^2+bx+c}dx = \int \sqrt{a[(x+\frac{b}{2a})^2 \pm k^2]}dx =$$

$$= \left[\frac{b^2-4ac}{4a^2} = \pm k^2; \quad x+\frac{b}{2a} = t; \quad dx = dt\right] = \int \sqrt{a(t^2 \pm k^2)}dt;$$

This integral is calculated using the following formula.

(A). $\int \sqrt{t^2+b}dt = \frac{t}{2}\sqrt{t^2+b} + \frac{b}{2}\ln\left|t+\sqrt{t^2+b}\right| + C$

(B). $\int \sqrt{a^2-t^2}dt = \frac{1}{2}\sqrt{a^2-t^2} + \frac{a^2}{2}\arcsin\frac{t}{a} + C$

An example. $\int \sqrt{x^2+2x+6}dx$ be counted.

To calculate this, after dividing a full square and specifying $t=x+1$, $b=5$, formula *(A)* is used. $x^2+2x+6=(x+1)^2+5$.

$$\int \sqrt{x^2+2x+6}dx = \int \sqrt{(x+1)^2+5}d(x+1) = \frac{x+1}{2}\sqrt{(x+1)^2+5} + \frac{5}{2}\ln\left|x+1+\sqrt{(x+1)^2+5}\right| + C$$

Integrating rational fractions. The ratio of two polynomials is called a rational fraction. $\frac{P_n(x)}{Q_m(x)}$. In this case, if "n<m", the rational fraction is called a correct rational fraction, if " n≥m ", the rational fraction is called an incorrect rational fraction. By dividing the image of such a fraction by its denominator, it is divided into whole and fractional parts. The fraction in this is a correct fraction. If

the denominator in a rational fraction, i.e $Q_m(x)=1$ then the fraction becomes a whole rational function. Integrating this is discussed above.

Now let's consider the integration of a proper rational fraction. First, we will see the integration of simple rational fractions. In general, a rational fraction is divided into simple fractions and then integrated. Simple rational fractions (sometimes called elementary fractions) have the following form.

A positive integer.

1. $\frac{A}{x-a}$ 2. $\frac{A}{(x-a)^k}$ Here $k \geq 2$

3. $\frac{Ax+B}{x^2+px+q}$. The root of the denominator consists of complex numbers, i.e $\frac{p^2}{4}-q \leq 0$

4. $\frac{Ax+B}{(x^2+px+q)^k}$. $k \geq 2$ is a positive integer.

Fractions of the form (1)-(4) are the simplest rational fractions. Now let's see the integration of these fractions.

$$1. \int \frac{A}{x-a}dx = A\ln|x-a|+C \qquad (C=const)$$

$$2. \int \frac{A}{(x-a)^k}dx = A\int (x-a)^{-k}dx = A\frac{(x-a)^{-k+1}}{-k+1}+C = \frac{A}{(1-k)(x-a)^{k-1}}+C$$

$$3. \int \frac{Ax+B}{x^2+px+q}dx = \int \frac{\frac{A}{2}(2x+p)+(B-\frac{Ap}{2})}{x^2+px+q}dx = \frac{A}{2}\int \frac{2x+p}{x^2+px+q}dx +$$

$$+(B-\frac{Ap}{2})\int \frac{dx}{x^2+px+q} = \begin{bmatrix} x^2+px+q=t & \int \frac{dt}{t}=\ln|t|+C \\ (2x+p)dx=dt & \end{bmatrix}$$

$$=\frac{A}{2}\ln|x^2+px+q|+(B-\frac{Ap}{2})\int\frac{dx}{(x+p/2)^2+(q-p^2/4)}=\frac{A}{2}\ln|x^2+px+q|+$$

$$+(B-\frac{Ap}{2})\int\frac{dx}{x^2+px+q}=\left[\begin{matrix}x^2+px+q=t \\ (2x+p)dx=dt\end{matrix}\quad \int\frac{dt}{t}=\ln|t|+C\right]=$$

$$=\frac{A}{2}\ln|x^2+px+q|+(B-\frac{Ap}{2})\int\frac{dx}{(x+p/2)^2+(q-p^2/4)}=\frac{A}{2}\ln|x^2+px+q|+$$
$$+\frac{2B-Ap}{\sqrt{4q-p^2}}arctg\frac{2x+p}{\sqrt{4q-p^2}}+C$$

$$4.\int\frac{Ax+B}{(x^2+px+q)^k}dx+\int\frac{A/2(2x+p)+B-Ap/2)}{(x^2+px+q)^k}dx=$$
$$=\frac{A}{2}\int\frac{2x+p}{(x^2+px+q)^k}dx+(B-\frac{Ap}{2})\int\frac{dx}{(x^2+px+q)^k}.$$

This is the first of the integrals $x^2+px+q=t$ with replacement $\int\frac{dt}{t}$ comes to an integral of the form This is according to the formula in the table of integrals $\frac{t^{-k+1}}{1-k}$ will be equal to.

If we subtract the full square from the denominator of the second integral, and $x+p/2=t$ performing the replacement and $q-p^2/4=m^2$ if we specify that, then $\int\frac{dt}{(t^2+m^2)^k}$ we come to the integral of the form By successively decreasing the degree of the denominator of this integral as follows $\int\frac{dt}{t^2+m^2}$ we bring to the integral of the form This is according to the formula in the table of integrals $\frac{1}{m}arctg\frac{t}{m}$ will be equal to,

i.e $\int\frac{dt}{(t^2+m^2)^k}=\frac{1}{m^2}\int\frac{(t^2+m^2)-t^2}{(t^2+m^2)^k}dt=\frac{1}{m^2}\int\frac{dt}{(t^2+m^2)^{k-1}}-\frac{1}{m^2}\int\frac{t^2}{(t^2+m^2)^k}dt \quad (*)$

however, $\int\frac{t^2dt}{(t^2+m^2)^k}=\int\frac{t\cdot tdt}{(t^2+m^2)^2}=-\frac{1}{2(k-1)}\int td(\frac{1}{(t^2+m^2)^{k-1}})$

Using the formula of integration by pieces, we make it look like this: $\int \frac{t^2 dt}{(t^2+m^2)^k} = -\frac{1}{2(k-1)}\left[t\cdot\frac{1}{(t^2+m^2)^{k-1}} - \int\frac{dt}{(t^2+m^2)^{k-1}}\right]$

Putting this in (*) we get:

$$\int\frac{dt}{(t^2+m^2)^k} = \frac{1}{m^2}\int\frac{dt}{(t^2+m^2)^{k-1}} + \frac{1}{m^2}\frac{1}{2(k-1)}\left[\frac{t}{(t^2+m^2)^{k-1}} - \int\frac{dt}{(t^2+m^2)^{k-1}}\right] =$$
$$= \frac{1}{2m^2(k-1)(t^2+m^2)^{k-1}} + \frac{2k-3}{2m^2(k-1)}\int\frac{dt}{(t^2+m^2)^{k-1}}$$

the degree of the denominator of the integral on the right is reduced by one.

So that $\int\frac{dt}{(t^2+m^2)^k}$ we reduced the indicator of the denominator of the integral by one. Using this method, the integral is given by repeating this operation $\int\frac{dt}{t^2+m^2}$ is brought to the integral of the form The rational fractional integral of the fourth form is calculated in this way.

The considered rational fractions were the simplest (elementary) rational fractions. Now, if a rational fraction of a different form is given, it is first expressed by the simplest rational fractions, and then integration is performed. That's why we see rational fractions represented by elementary fractions. Proper fractions of the following form can always be represented by elementary fractions:

$\frac{A}{(x-a)^m}$; $\frac{Mx+N}{(x^2+px+q)^n}$. (m, n are positive integers).

Right for us $\frac{P_n(x)}{Q_m(x)}$ let a rational fraction be given. This fraction is divided into elementary fractions as follows:

a) $Q_m(x)$ the denominator is divided into the multipliers. It can have linear and quadratic multipliers.

$Q(x)=a_o(x-a_1)^m...(x-a_{m-k})^k(x^2+px+q)^n...(x^2+p_{n-r}x+q_{n-r})^r$.

b) The given fraction is divided into elementary fractions in a schematic view:

$$\frac{P_n(x)}{Q_m(x)}=\frac{A_1}{x-a}+\frac{A_2}{(x-a)^2}+...+\frac{A_m}{(x-a)^m}+\frac{B_1}{x-b}+\frac{B_2}{(x-b)^2}+\frac{B_k}{(x-b)^k}+\frac{M_1x+N_1}{x^2+px+q}+$$

$$+\frac{M_2x+N_2}{(x^2+px+q)^2}+...+\frac{M_nx+N_n}{(x^2+px+q)^n}+...+\frac{C_1x+D_1}{x^2+cx+d}+...+\frac{C_rx+D_r}{(x^2+cx+d)^r} \quad (*)$$

here A_m, ..., B_k, ..., M_n, ..., N_n, ..., C_r, ..., D_r are fixed numbers. How many multiple roots *Q(x)* has, the number of elementary fractions in the scheme is the same.

c) We save the fraction from the denominator by multiplying both sides of the resulting equality (*) by *Q(x)*.

g) Then we create a system of equations by equating the coefficients in front of the same levels of *"x"* on both sides of the resulting equation.

This is the number of equations in the system $A_1,...B_1,...M_1,...N_1,...C_1,...D_1,...$, should be equal to the number of unknowns.

d) The resulting system of equations is solved, the unknown coefficients are found, and they are put in (*) and both sides are multiplied by dx and integrated. The resulting elementary fractions consist of fractions of the form (1)-(4).

An example. $\int \frac{3x^2+8}{x^3+4x^2+4x}dx$ be counted.

Solving. We divide the denominator of the fraction under the integral into the multipliers. $x^3+4x^2+4x=x(x^2+4x+4)=x(x+2)^2$

Now let's expand the given fraction into elementary fractions using (*):

$$\frac{3x^2+8}{x(x+2)^2}=\frac{A}{x}+\frac{B}{x+2}+\frac{C}{(x+2)^2} \qquad (**)$$

multiply both sides of this by *x(x+2)2.*

$3x^2+8=A(x+2)^2+Bx(x+2)+Cx=(A+B)x^2+(4A+2B+C)x+4A$

Now, by equating the coefficients of *"x"* in front of the same levels, we create a system of equations:

$$\begin{cases} A+B=3 \\ 4A+2B+C=0 \\ 4A=8 \end{cases}$$

By removing this system, *A=2, B=1, C=-10* we find. Then we put these in (**).

$$\frac{3x^2+8}{x(x+2)^2}=\frac{2}{x}+\frac{1}{x+2}-\frac{10}{(x+2)^2}.$$

Multiply both sides by *"dx"* and then integrate:

$$\int\frac{3x^2+8}{x^3+4x^2+4x}dx=\int[\frac{2}{x}+\frac{1}{x+2}-\frac{10}{(x+2)^2}]dx=2\int\frac{dx}{x}+\int\frac{dx}{x+2}-10\int(x+2)^{-2}d(x+2)=$$

$=2ln|x|+ln|x+2|+10/(x+2)+C$

Integrating some trigonometric functions.

1. Integrals of this form are given

$$\int sin\,n\,x\,cos\,m\,x dx, \quad \int cos\,n\,x\,cos\,m\,x dx, \quad \int sin\,n\,x\,sin\,m\,x dx.$$

$|n| \neq |m|$

To calculate the integral of these transcendental functions, the given integrals can be integrated using the following formulas.

$$sin\,n\,x\,cos\,m\,x = \frac{1}{2}[sin(n+m)x + sin(n-m)x]$$

$$cos\,n\,x\,cos\,m\,x = \frac{1}{2}[cos(n+m)x + cos(n-m)x]$$

$$sin\,n\,x\,sin\,m\,x = \frac{1}{2}[cos(n-m)x - cos(n+m)x]$$

Example 1.

$$sin\,5\,x\,cos\,3\,x dx = \frac{1}{2}\int(cos\,2\,x - cos\,8\,x)dx = \frac{1}{4}sin\,2\,x - \frac{1}{16}sin\,8\,x + C$$

Example 2.

$$sin\,4\,x\,cos\,2\,x dx = \frac{1}{2}\int(sin\,6x + sin\,2\,x)dx = -\frac{1}{12}cos\,6\,x - \frac{1}{4}cos\,2\,x + C$$

Example 3.

$$cos\,2\,x\,cos\,8\,x dx = \frac{1}{2}\int(cos\,6\,x + cos\,10x)dx = \frac{1}{12}sin\,6\,x + \frac{1}{20}sin\,10x + C$$

2. $J = \int sin^p\,cos^q\,x\,dx$ integral is given (p and q are integers).

A) If at least one of the integers *“p”* and *“q”* is odd, then *q=2K+1* $J = \int sin^p\,cos^{2K+1}\,x\,dx = \int sin^p\,cos^{2K}\,x\,cos\,x\,dx$ will be.

Sinx=t we define with *dt=cosxdx* will be.

So, $J = \int t^p(1-t^2)^k dt$ It comes to a rational function with respect to *"t"*.

Example 4.

$$\int \frac{cos^3 x}{sin^4 x}dx = \int \frac{cos^2 x\,cos\,x}{sin^4 x}dx = \begin{vmatrix} sin\,x = t \\ cos\,x\,dx = dt \end{vmatrix} = \int \frac{(1-t^2)}{t^4}dt =$$

$$= \int \frac{dt}{t^4} - \int \frac{dt}{t^2} = -\frac{1}{4t^3} + \frac{1}{t} + C = \frac{-1}{4\sin^3 x} + \frac{1}{\sin x} + C$$

B) Let the numbers *"p"* and *"q"* be positive and even numbers. *q=2K*, *q=2S* then this

$$\cos^2 x = \frac{1 + \cos 2x}{\mathbf{2}}$$

and

$$\sin^2 x = \frac{1 + \cos 2x}{\mathbf{2}}$$

we use formulas. Using these formulas, the degree of sine and cosine decreases by a factor of 2.

$$\int \sin^p \cos^q x\, dx = \int \sin^{2K} \cos^{2S} x\, dx = \int \left(\frac{1-\cos 2x}{2}\right)^K \left(\frac{1+\cos 2x}{2}\right)^S dx.$$

Example 5.

$$\int \sin^2 x \cos^2 x\, dx = \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx = \frac{1}{4}\int (1-\cos^2 2x)dx =$$

$$= \frac{1}{4}\int \frac{1-\cos 4x}{2} dx = \frac{1}{8}\int dx - \frac{1}{8}\int \cos 4x dx = \frac{x}{8} - \frac{1}{32}\sin 4x + C$$

C) If the numbers *"p"* and *"q"* are even, and one of them is negative, the other is replaced.

Example 6.

$$\int \sin^2 x \cos^{-4} x\, dx = \int \frac{\sin^2 x}{\cos^4 x} dx = \int tg^2 x \frac{dx}{\cos^2 x} = \left| \begin{matrix} tgx = t \\ \frac{dx}{\cos^2 x} = dt \end{matrix} \right| =$$

$$= \int t^2 dt = \frac{t^3}{3} + C = \frac{tg^3 x}{3} + C;$$

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