SOLVING GEOMETRY PROBLEMS OF TRIANGLES USING COMPLEX NUMBERS

Musoyeva Feruza

Teacher of Non-state educational institution

"Mamun University"

In this article, the application of complex numbers for triangular problems is considered.

We may solve many problems by using complex numbers in geometry. Especially, we can use from complex numbers for solving problems about similar triangles and their properties.

ABC and $A_1B_1C_1$ triangles which similar and the same orientation be given us.

If $A_1B_1 = kAB$ $A_1B_1 = kAC$ and $\angle B_1A_1C_1 = \angle BAC$ (angles orientate) then ABC and $A_1B_1C_1$ triangles can be similar and the same orientation. [1]

We can write the following equals with complex numbers:

$$|a_1 - b_1| = k|a - b|, \quad |a_1 - c_1| = k|a - c| \quad \arg\frac{c_1 - a_1}{b_1 - a_1} = \arg\frac{c - a}{b - a}.$$

Both of equal

$$\frac{|c_1 - a_1|}{|b_1 - a_1|} = \frac{|c - a|}{|b - a|} \quad \text{va} \quad \arg\frac{c_1 - a_1}{b_1 - a_1} = \arg\frac{c - a}{b - a}$$

has the same equivalent.

$$\frac{c_1 - a_1}{b_1 - a_1} = \frac{c - a}{b - a} \quad \text{or} \quad \frac{c_1 - a_1}{c - a} = \frac{b_1 - a_1}{b - a} = s \qquad (1)$$

[1] Я.П.Понарин. Алгебра комплексных чисел в Геометрических задачах. Москва -2004 15-25Б

Here s - complex number, $|s| = k$ - similarity coefficient.

If a private condition s -is a real number, then $s = \frac{c_1 - a_1}{c - a} = \frac{\overline{c_1} - \overline{a_1}}{\overline{c} - \overline{a}}$

and according to $(a - b)(\overline{c} - \overline{d}) = (\overline{a} - \overline{b})(c - d)$, $AC \,||\, A_1C_1$ ($||$-parallelism sign).[2]

As if because of this $AB \,||\, A_1B_1$ and $BC \,||\, B_1C_1$. According to this reason, come out that ABC and $A_1B_1C_1$ the triangular gomotetik.

(1) the fit is necessary and sufficient condition when ABC and $A_1B_1C_1$ triangles are similar and the same orientation. It can give symmetrically form

$$\begin{vmatrix} a & a_1 & 1 \\ b & b_1 & 1 \\ c & c_1 & 1 \end{vmatrix} = 0 \qquad (2)$$

or

$$ab_1 + bc_1 + ca_1 = ba_1 + cb_1 + ac_1 \qquad (3)$$

ABC and $A_1B_1C_1$ triangles are similar and the opposite orientation (similarity of the second kind).

If only $A_1B_1 = kAB$, $A_1C_1 = kAC$ and $\text{P}B_1A_1C_1 = -\text{P}BAC$ we shall write this equal as follows.

$$\arg\frac{c_1 - a_1}{b_1 - a_1} = -\arg\frac{c - a}{b - a} = \arg\frac{\overline{c} - \overline{a}}{\overline{b} - \overline{a}}.$$

Both of equality are the same equivalent

$$\frac{|c_1 - a_1|}{|b_1 - a_1|} = \frac{|\overline{c} - \overline{a}|}{|\overline{b} - \overline{a}|} \text{ va } \arg\frac{c_1 - a_1}{b_1 - a_1} = \arg\frac{\overline{c} - \overline{a}}{\overline{b} - \overline{a}}.$$

[2] А.Г.Курош.Олий алгебра курси.Т.Ўкитувчи.1976 й. 464 б.

$$\frac{c_1 - a_1}{b_1 - a_1} = \frac{\bar{c} - \bar{a}}{\bar{b} - \bar{a}}, \quad \frac{c_1 - a_1}{\bar{c} - \bar{a}} = \frac{b_1 - a_1}{\bar{b} - \bar{a}} = s . \qquad (4)$$

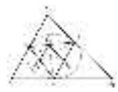
Here s -complex number, $|s| = k$ - similarity coefficient. (4) the fit is necessary and sufficient condition when ABC and $A_1B_1C_1$ triangles are similar and the same orientation.It can give symmetrically form

$$\begin{vmatrix} \bar{a} & a_1 & 1 \\ \bar{b} & b_1 & 1 \\ \bar{c} & c_1 & 1 \end{vmatrix} = 0 \qquad (5)$$

1. If triangle ABC is form by connecting the attemt points of drawn inside circle with triangle $A_1B_1C_1$, and triangle $A_2B_2C_2$ is form by connecting the points of intersection based on heights with the sides of the triangle ABC, then prove that triangles $A_1B_1C_1$ and $A_2B_2C_2$ are homototic.(picture 1)

Prove. As usual, we shall take as unique. The following equals come out by $\frac{1}{z} = \frac{1}{2}(\frac{1}{a} + \frac{1}{b})$ and $z = \frac{1}{2}(a + b + c - \frac{bc}{a})$ formulas

$$a_1 = \frac{2bc}{b + c}, \quad a_2 = \frac{1}{2}(a + b + c - \frac{bc}{a}),$$



$$b_1 = \frac{2ac}{a + c}, \quad b_2 = \frac{1}{2}(a + b + c - \frac{ac}{b}),$$

$$c_1 = \frac{2ab}{a + b}, \quad c_2 = \frac{1}{2}(a + b + c - \frac{ab}{c}).$$

Let us check the satisfaction of (1) mark (sign).

$$\frac{a_1 - b_1}{a_2 - b_2} = \frac{a_1 - c_1}{a_2 - c_2} = \frac{-4abc}{(a + b)(b + c)(c + a)} = s ,$$

here $s = \bar{s}$, that is s -real number. Thus, $A_1B_1C_1$ and $A_2B_2C_2$ triangles are homototic.[3]

2. Two ABC and $A_1B_1C_1$ triangles which are congluator and with the same orientation. If $\overrightarrow{MA_0}, \overrightarrow{MB_0}, \overrightarrow{MC_0}$ vectors are equal to $\overrightarrow{AA_1}, \overrightarrow{BB_1}, \overrightarrow{CC_1}$ vectors for M point in the plain, you must prove that $A_0B_0C_0$ triangle is similar and the same orientation to devoted triangles.

Solve. we can say $m = 0$. Thus,

$$a_0 = a_1 - a, b_0 = b_1 - b, c_0 = c_1 - c,$$

from $b_0 - a_0 = b_1 - a_1 - (b - a), c_0 - a_0 = c_1 - a_1 - (c - a)$, and so

$$\frac{b_0 - a_0}{b - a} = \frac{b_1 - a_1}{b - a} - 1, \quad \frac{c_0 - a_0}{c - a} = \frac{c_1 - a_1}{c - a} - 1$$

according to (1) equality and ploblem condition , we can take the followings $\frac{b_1 - a_1}{b - a} = \frac{c_1 - a_1}{c - a}$.Thus, $\frac{b_0 - a_0}{b - a} = \frac{c_0 - a_0}{c - a}$ is arise. According to (1) equality again, $A_0B_0C_0$ and ABC triangles are similar and the same orientation.[4]

3. Two ABC and $A_1B_1C_1$ triangles which are congluator and with the same orientation and drawn inside circle. Here, it must proved that the ends of the triangle is similar to the triangles at the points of intersection of the lines CA and C_1A_1, AB and A_1B_1.

Prove. It will be $z\bar{z} = 1$ circle equation. When $A_1B_1C_1$ triangular ends turning at an angle, it can servisethe images of ABC triangular which are $\arg a$, $|a| = 1$.

[3] Д.К.Фаддеев и И.С.Соминский. Сборник задач по высшей алгебре. М.Наука. 1976г.304с.
[4] . Л.Б.Шнеперман. Курс алгебры и теории чисел в задачах и упражнениях. I и II часть. Минск» Выш.шк.» 1987 г.272с.

So $a_1 = a a, b_1 = a b, c_1 = a c$. If $A_2B_2C_2$ - BC and B_1C_1, CA and C_1A_1, AB and A_1B_1 lines points of intersection in a straight line. Then, accordingly

When we based on $\bar{z} = \dfrac{(a+b) - (c+d)}{ab - cd}$, the following equals follows .

$$\bar{a}_2 = \frac{b + c - (ab + ac)}{bc - a^2 bc} = \frac{b + c}{bc\left(1 + a\right)},$$

from this equals $a_2 = \dfrac{b + c}{1 + \bar{a}}$. In similar way $b_2 = \dfrac{a + c}{1 + \bar{a}}$, $c_2 = \dfrac{a + b}{1 + \bar{a}}$. When this conditions,

$$\begin{vmatrix} a & a_2 & 1 \\ b & b_2 & 1 \\ c & c_2 & 1 \end{vmatrix} = \frac{1}{1 + \bar{a}}, \begin{vmatrix} a & b + c & 1 \\ b & c + a & 1 \\ c & a + b & 1 \end{vmatrix} = 0$$

The solve of determinant arise directly.

References

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