METHODS OF SOLVING ECONOMIC PROBLEMS USING DIFFERENTIAL EQUATIONS

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Suppose, any product is sold at a price p, let the function $Q(t)$ is produced changes the volume of manufacture product over time t, in that case income equal to $pQ(t)$ is obtained.

Let’s say that part of the income is used to invest in production. That is

$$I(t) = mpQ(t) \tag{1}$$

m-investmest norm, constant and $0 < m < 1$.

If we based on the assumption that the market is supplied suffi ciently and that product is sold fully, the rate of manufacturing product will bring to increase (accelerator) again. The rate of manufacturing is proportional to increase of investment.

$$Q' = lI(t) \tag{2}$$

where $1/l$ - accelerator (increase) norm.Substituting formula (1) into (2)

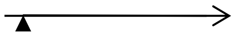
$$Q' = kQ, k = lmp \tag{3}$$

we obtain the differential equation. (2.1.3) is separate variables differential equation. $Q = Ce^{kt}$ is the general solution of this equation, where C-voluntary constant.

Suppose, Q_0 is the volume of manufacturing product at initial time $t = t_0$. In that case, constant coefficient C can be defined from this condition: $Q_0 = Ce^{kt_0}$, it is follows from this $C = Q_0 e^{-kt_0}$. As a result, we find the solution of the Cauchy ploblem for equation (3).

$$Q = Q_0 e^{k(t-t_0)} \tag{4}$$

For simplicity, $P(Q) = a - bQ, a > 0, b > 0$ let we consider the case which the demand function is the linear function of manufacturing function[1].



In that case equation $Q' = aP(Q)Q$

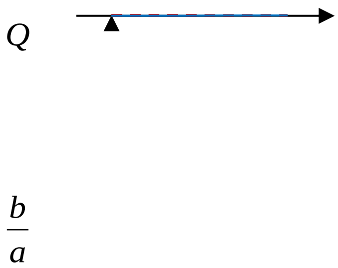
$$Q' = \alpha(a - bQ)Q \tag{5}$$

will look like. If $Q = 0$ or $Q = \frac{b}{a}$, in that case $Q' = 0$.

Thus,

$$Q'' = \alpha Q'(a - 2bQ). \tag{6}$$

Also,when $Q < \frac{b}{2a}$, $Q'' > 0$ and $Q > \frac{b}{2a}$ in that case $Q'' > 0$.



[1] W.A. Brock, A.G. Malliaris. Differential equations, Stability and Chaos in Dynamic Economics, North Holland, 1989

$\frac{b}{2a}$ $\qquad\qquad$ t

The inflection point of graph function $Q = Q(t)$ is $t = Q = a/2b$. In that case, solution $Q(t)$ can find clearly. We separating the variables in the equation (4)

$$\frac{dQ}{Q(a-bQ)} = \alpha dt$$

or

$$\frac{1}{a}\left(\frac{1}{Q} + \frac{b}{a-bQ}\right)dQ = \alpha dt.$$

We integrate this relation

$$\frac{Q}{a-by} = Ce^{\alpha at}.$$

According to above equal

$$Q(t) = \frac{aCe^{\alpha at}}{1+bCe^{\alpha at}}$$

The graph of this function which is given in the drawing is called *logistic curve line* of differential equation (4). Example 1. The rate of depreciation of equipment which as a result of wear is proportional to the current cost for whole time. The initial cost is A_0. How much will the equipment cost in t years?Let A_t be the cost of equipment at time t . The change in cost (depreciation) is expressed as $A_0 - A_t$ diversity. The rate of depreciation

$$\frac{d}{dt}(A_0 - A_t)$$

is proportional to the actual value at a given time A_t. We obtain an equation

$$\frac{d(A_0 - A_t)}{dt} = kA_t$$

with the initial condition $A_t\big|_{t=0} = A_0$.

Solving it, we get the following solution

$$\frac{dA_1}{dt} = kA_t; \qquad \int\frac{dA_1}{A_t} = -\int kdt; \qquad ln|A_t| = -kt + ln|C|;$$

$$ln\left|\frac{A_1}{C}\right| = -kt; \qquad \frac{A_1}{C} = e^{-kt}; \qquad A_t = Ce^{-kt}.$$

voluntary derivative for determinate S, we use the initial condition $A_t = A_0$ for $t = 0$ $: A_0 = Ce^{-k\times 0}$, $C = A_0$, $A_t = A_0e^{-kt}$. The obtained partial solution is answer of this problem.[2]

Example 2. Let the supply and demand for a product be determined by the corresponding reciprocal ratio

$$D = 4p' - 2p + 39, \quad S = 44p' + 2p - 1,$$

where p is the price of the product; p' - the trend of price formation (the price of pruduction during the time). Let we also assume that at the initial moment of time, the price p 1 per unit of money. Based on the requirement that the corresponds to the supply, we find the law of price change depending on time.

The following equation must be satisfied for the demand to match the supply

$$4p' - 2p + 39 = 44p' + 2p - 1.$$

from here

$$10p' + p - 10 = 0.$$

We obtain differential equation with separable variables:

[2] P.A. Samuelson. Foundations of Economic Analysis. Harvard University Press, Cambridge, 1947

$$-10\frac{dp}{dt} = p-10,\ \frac{dp}{p-10} = -\frac{dt}{10},\ ln|p-10| = -\frac{t}{10} + ln|C|,\ ln\left|\frac{p-10}{C}\right| = -\frac{t}{10},$$

$$\frac{p-10}{C} = e^{-0,1t},\ p = Ce^{-0,1t} + 10.$$

Note that $p|_{t=0} = 1,$ then

$$1 = C + 10;\ C = -9;\ p = -9e^{-0.1t} + 10.$$

Thus, the price must change in according to the obtainable formula for the balance between supply and demand to remain equal[3].

Example 3. Let only x buyers know from the number of potential buyers at the time t about the products sold by commercial establishments. After posting advertising, the rate of change cost product is proportional the number of buyers who know about the brand too and the number of buyers who do not know about the brand too. Generally, $N/$ people was known about product at the initial time $t = 0$, (the time is counted after advertising), γ - the specified number. Find the law of time-dependent change of x the number of custumers who know about the products.

According to the condition, we obtain the following form equation

$$\frac{dx}{dt} = kx(N-x).$$

for determining $x = x(t)$ equation. Where $\frac{dx}{dt}$ is the rate of change in the number of buyers who know about the product;

x is the number of people who know about the product;

[3] R.F. Harrod, An Essay in Dynamic Theory, The Economic Journal, 49(193) (1939), 14–33

$N-x$ is the number of people who do not know about the product at time t ;

k is a positive proportionality coefficient.

Initial condition: $x|_{t=0} = N/\gamma$. We solve the differential equation, which is an equation with separable variables:

$$\frac{dx}{x(N-x)} = k\,dt.$$

As a result of integration, we have

$$\frac{1}{N}\ln\left|\frac{x}{N-x}\right| = kt + C.$$

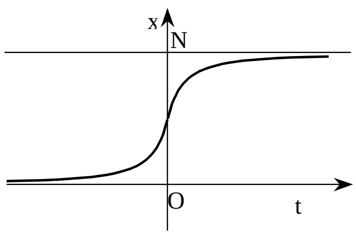
Assuming $NC = C_{C1}$, we arrive at the equality

$$\frac{x}{N-x} = Ae^{Nkt},$$

where $A = e^{C_1}$. Let's solve the last equation with respect to x:

$$x = N\frac{A\cdot e^{Nkt}}{A\cdot e^{Nkt}+1} = \frac{N}{1+p\cdot e^{-Nkt}},$$

where $p = 1/A$. The resulting equation is called the equation of the logistic curved line.



Consider the initial conditions: $\frac{N}{\gamma} = \frac{N}{1+p}$; $1+p=\gamma;\ \ p=\gamma-1$. Then $x = \frac{N}{1+(\gamma-1)e^{-Nkt}}$ - the law of changes in the number of buyers as a function of time t. In particular, for $\gamma = 2$, we obtain $x = \dfrac{N}{1+e^{-Nkt}}$. The figure show a logistic curve line in $\gamma = 2$.[4]

References

1. W.A. Brock, A.G. Malliaris. Differential equations, Stability and Chaos in Dynamic Economics, North Holland, 1989

2. P.A. Samuelson. Foundations of Economic Analysis. Harvard University Press, Cambridge, 1947

3. R.F. Harrod, An Essay in Dynamic Theory, The Economic Journal, 49(193) (1939), 14–33

4. P.A. Samuelson, The stability of equilibrium: Comparative statics and dynamics, Econometrica, 9(2) (1941), 97–120

[4] P.A. Samuelson, The stability of equilibrium: Comparative statics and dynamics, Econometrica, 9(2) (1941), 97–120