BIR FRIDRIXS MODEL OPERATORINING MUHIM SPEKTRDAN TASHQARIDAGI XOS QIYMATI HAQIDA

Kurbonova Yulduz Rasulovna

Buxoro davlat universiteti, Matematika (yo'nalishlar bo'yicha) mutaxasisligi magistranti

Annotatsiya: Bu ishda Fridrixs model operator qaralgan bo'lib muhim spektrdan tashqaridagi xos qiymatlarining mavjudligi o'rganilgan.

Kalit so'zlar: gilbert fazosi, xos qiymat, spektr, yo'qotish operatori, paydo bo'lish operatori, Shroedinger operatori, Xaar o'lchovi, ikki zarrachali operator, tasir potensiali, operator rangi, maks prinsipi.

Asosiy matn. $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1$ Gilbert fazosi. Bunda $\mathcal{H}_0 = C^1$ kompleks sonlar gilbert fazosi va $\mathcal{H}_1 = L^{2,e}(\mathbb{T}^1)$ bilan $\mathbb{T}^1 = (-\pi, \pi]$ da modulining kvadrati bilan integrallanuvchi juft funksiyalar gilbert fazosini belgilaymiz.

Bir o'lchamli $\mathbb{Z}$ panjaradagi soni ikkitadan oshmaydigan zarrachalardan iborat sistemaga mos $H_{\gamma\mu}(k), k \in \mathbb{T}$ operator oilasini qaraymiz. Bunda zarrachalar nafaqat o'zaro ta'sir potensiali V_μ orqali, balki paydo bo'lishi yoki yo'qotish operatorlari C_γ va $C_\gamma{}^*$ γ>0 yordamida ta'sirlashadi:

$$H_{\gamma\mu}(k) = \begin{pmatrix} E(k) & C_\gamma{}^* \\ C_\gamma & H_\mu(k) \end{pmatrix},$$

E(k), k=∈ $\mathbb{T}^1$- operator $\mathcal{H}_0$ Gilbert fazosida songa kopaytirish operatori bo'lib quyidagi formula yordamida aniqlanadi:

$$\mathrm{E}(k) f_0 = \varepsilon(k) f_0, \ f_0 \in \mathcal{H}_0,$$

bunda $\varepsilon(k) = -2(1 - cosk)$.

$C_\gamma{}^*: \mathcal{H}_1 \to \mathcal{H}_0$ va $C_\gamma: \mathcal{H}_0 \to \mathcal{H}_1$ operatorlar mos ravishda yo'q qiluvchi va paydo qiluvchi operatorlar bo'lib , quyidagicha aniqlanadi:

$$C_\gamma{}^* f_1 = \gamma(f_1, \alpha_0)_{\mathcal{H}_1}, \;\; C_\gamma f_0 = \gamma\alpha_0(f_0, \alpha_0)_{\mathcal{H}_1}$$

$H_\mu(k)$, $=\in \mathbb{T}^1$ operator esa $\mathcal{H}_1$ Gilbert fazosidagi nuqtada ta'sirlashuvchi ikkita bir xil zarrachalar (bozonda) sistemasi Hamiltoniga mos ikki zarrachali diskret Shroedinger operatori bo'lsin:

$$H_\mu(k) = H_0(k) + V_\mu,$$

bunda $H_\mu(k) - \varepsilon_k(\cdot)$ funksiyaga ko'paytirish operatori:

$H_{\lambda\mu}(k)$, $k \in \mathbb{T}^1$, operator $\mathcal{H}_1$ Gilbert fazosida aniqlangan nuqtada va bir qadamda tasirlashuvchi ikki zarrachali diskret Shroedinger operatori bo'lib quyidagi formula yordamida aniqlangan :

$$H_{\mu\lambda}(k)\, f_1(q) = \varepsilon_k(q) f_1(q) + \int_{\mathbb{T}^1} (\mu + \lambda \cos s \cos q)\, f_1\,(s) d\eta,$$

bunda $\varepsilon_k(q) = \varepsilon\left(\frac{k}{2} - q\right) + \varepsilon\left(\frac{k}{2} + q\right)$, $d\eta = d\eta(q)$ Xaar o'lchovi , ya'ni $d\eta = \frac{dq}{(2\pi)^1}$

$H_{\gamma\mu\lambda}(k)$, $k \in \mathbb{T}^d$, $\mu \leq 0$ va $\gamma, \lambda \in [0, +\infty)$ operator $\mathcal{H}$ Gilbert fazosida quyidagi ormula yordamida aniqlangan:

$$H_{\gamma\mu\lambda}\,(k)\begin{pmatrix} f_0 \\ f_1(q) \end{pmatrix} = \begin{pmatrix} E(k) f_0 + C_\gamma^* f_0 \\ C_\gamma^* f_0 + H_{\mu\lambda}(k) f_1(q) \end{pmatrix},$$

bunda $C_\gamma^* f_1 = \gamma(f_1, 1)_{\mathcal{H}_1}$ (mos ravishda. $C_\gamma^* f_0 = \gamma(f_0, 1)_{\mathcal{H}_0}$) paydo qiluvchi va (mos ravishda .yo'qotuvchi) operatorlar [2-4].

Теорема. b) Shunday λ va γ, $\mu \in [0, +\infty)$, $k \in \mathbb{T}^1$ lar mavjudki, $H_{\gamma\mu\lambda}(k)$ operator ko'pi bilan ikkita turli ishorali xos qiymatlarga ega:

a) Shunday λ va γ, $\mu \in [0, +\infty)$, $k \in \mathbb{T}^1$ lar mavjudki, $H_{\gamma\mu\lambda}(k)$ operator faqat manfiy xos qiymatga ega.

b) Shunday λ va γ, $\mu \in [0, +\infty)$, $k \in \mathbb{T}^1$ lar mavjudki, $H_{\gamma\mu\lambda}(k)$ operator faqat musbat xos qiymatga ega.

Tasdiq 1. $H_{\gamma\mu}(k)$ va $H_{0\mu}(k)$ operatorlarning ayirmasining rangi ikkiga teng bo'lganligi uchun mini maks prinsipiga asosan $H_{\gamma\mu}(k,z)$ operatorning xos qiymatlari soni ikkitadan oshmaydi.

Tasdiq 2. Ixtiyoriy , $\mu \in [0,+\infty)$ va $k \in (\pi,\pi)$ lar uchun

$$\Delta_{\gamma\mu}(k,z)=C_{-\frac{1}{2}}(\gamma,\mu,k)(\varepsilon_{min}(k)-z)^{-\frac{1}{2}}+C_0(\gamma,\mu,k)+O(\varepsilon_{min}(k)-z)^{-\frac{1}{2}}$$

$z\to \varepsilon_{min}(k)$-tenglik o'rinli.

Bu yerda

$$C_{-\frac{1}{2}}(\gamma,\mu,k)=\frac{F_{\gamma\mu}(k)}{4cos^{\frac{3}{2}}\frac{k}{2}},\quad F_{\gamma\mu}(k)=-2\gamma^2(cos\frac{k}{2})+2\mu cos\frac{k}{2}$$

$$C_0(\gamma,\mu,k)=\frac{(\varepsilon_{min}(k)-z)^{-\frac{1}{2}}}{4cos^{\frac{3}{2}}\frac{k}{2}}[4(1-cos\frac{k}{2})cos^2\frac{k}{2}]$$

Teorema. a) $k\epsilon\mathbb{T}$ va $\frac{\gamma^2}{\mu}\le \varepsilon(k)-\varepsilon_{min}(k)=2cos\frac{k}{2}(1-cos\frac{k}{2})$ bo'lsin. U holda $H_{\gamma\mu}(k)$ operatorning muhim spektrdan tashqararida yagona $E^1{}_{\gamma\mu}(k)$ xos qiymatga ega va bu xos qiymat quyidagi

$$\varepsilon_{min}(k) < E^1{}_{\gamma\mu}(k)$$

tengsizlikni qanoatlantiradi.

b) $k\epsilon\mathbb{T}$ va $\frac{\gamma^2}{\mu}>\varepsilon(k)-\varepsilon_{min}(k)=2cos\frac{k}{2}(1-cos\frac{k}{2})$ bo'lsin. U holda $H_{\gamma\mu}(k)$ operatorning muhim spektrdan tashqarida ikkita $E^2{}_{\gamma\mu}(k)$ va $E^1{}_{\gamma\mu}(k)$ xos qiymatlarga ega va bu xos qiymatlar quyidagi

$$E^2{}_{\gamma\mu}(k)<\varepsilon_{min}(k)\le\varepsilon_{max}(k)<E^1{}_{\gamma\mu}(k)$$

tengsizlikni qanoatlantiradi.

Foydalanilgan adabiyotlar

1. С.Н.Лакаев, Ш.М.Латипов. Числа связанных состояний двухканальной молекулярной-резонансной модели, УзМЖ 2011,№ 3 стр 98-113.

2. С.Н.Лакаев, Ш.М.Латипов. О существование и аналитичности Собственных значений двухканальной молекулярно-резонансной модели, ТМФ 169(2011) №3 стр. 1657-1666.

3. Расулов Х.Р. Аналог задачи Трикоми для квазилинейного уравнения смешанного типа с двумя линиями вырождения // Вестн. Сам. гос. техн. ун-та. Сер. Физ.-мат. науки, 2022. Т. 26, № 4.

4. Rasulov X.R. Qualitative analysis of strictly non-Volterra quadratic dynamical systems with continuous time // Communications in Mathematics, 30 (2022), no. 1, pp. 239-250.