$h_{\mu}(k)$ $k \in T^d$ OPERATORNING MUHIM SPEKTRI VA SPEKTRAL XOSSALARI

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Maqolada ikki kanalli molekulyar rezonansli model operatorning xos qiymatlari qaralgan bo‘lib, $h_{\mu}(k)$ $k \in T^d$ operatorning muhim spektri va spektral xossalari o‘rganilgan. v_{μ} – rangi birdan oshmaydigan integral operator bo‘lganligi uchun muhim spektr turg‘unligi haqidagi Veyl teoremasiga ko‘ra, $h_{\mu}(k)$ operatorning muhim spektri $\sigma_{ess}(h_{\mu}(k))$, $\mu \in \mathrm{R}$ ga bog‘liqsiz va $h_0(k)$ operatorning spektri $\sigma(h_0(k))$ bilan ustma-ust tushadi. Shunday qilib, quyidagi tengliklar o‘rinli [1]:

$$\sigma_{ess}(h_{\mu}(k)) = \sigma(h_0(k)) = [m(k), M(k)],$$

bunda

$$m(k) = \min_{p \in T^d} \varepsilon_k(p) = 2 \sum_{i=1}^{d} \left(1 - \cos\frac{k_i}{2}\right) \geq 0\,,$$

$$M(k) = \max_{p \in T^d} \varepsilon_k(p) = 2 \sum_{i=1}^{d} \left(1 + \cos\frac{k_i}{2}\right) \leq 4d\,.$$

Ta’kidlab o‘tamizki, ixtiyoriy $\mu \geq 0$ da v_{μ} $L_2^e(T^d)$ fazoda nomanfiy operator bo‘ladi (mos ravishda $\mu \leq 0$ *bo*‘lganda musbat operator bo‘ladi). $z \in \mathrm{C} \setminus [m(k), M(k)]$ larda aniqlangan funksiya ($h_{\mu}(k)$ operatorga mos Fredgolm determinanti) quyidagi formula yordamida aniqlangan [2] :

$$\Delta(\mu, k, z) = 1 - \frac{\mu}{2\pi} \int_{k \in T^d} \frac{dp}{\varepsilon_k(p) - z} = 1 - a(k, z)$$

O‘z-o‘ziga qo‘shma $h_\mu(k)$ operatorning xos qiymatlari va $\Delta(\mu, k, z)$ funksiyaning nollari orasidagi bog‘liqlikni quyidagi lemma o’rnatadi:

Lemma 1. Ixtiyoriy $\mu \in R$ *va* $k \in T^d$ lar uchun $z \in \mathbb{C} \setminus [m(k), M(k)]$ soni $h_\mu(k)$ operatorning xos qiymati uchun $\Delta(\mu, k, z) = 0$ bo‘lishi zarur va yetarli.

Shuni ta’kidlash joizki $h_\mu(k)$, $k \in T^d$ operatorning muhim spektri $\mu \in R0$ ga bog‘liqsiz $[m(k), M(k)]$ kesmadan iborat va bu operatorning $\mu \geq 0$ bo‘lganda $[m(k), M(k)]$ kesmadan o‘ng tomonda xos qiymatga ega emas hamda mos ravishda $\mu \leq 0$ bo‘lganda $[m(k), M(k)]$ kesmadan chap tomonda xos qiymatga ega emas [3].

Quyidagi belgilashlarni kiritamiz :

$$\mu_{max}^0(k) = -\left(\int_{T^d} \frac{dp}{-\varepsilon_k(p) + M(k)}\right)^{-1}$$

$$\mu_{min}^0(k) = \left(\int_{T^d} \frac{dp}{\varepsilon_k(p) - m(k)})\right)^{-1}$$

Bundan natija sifatida quyidagi teoremani keltiramiz [1-3]:

Teorema 1. *a) Itiyoriy* $\mu > 0$ *va* $k \in T^d$ *lar uchun* $h_\mu(k)$ *operator yagona* $\zeta_1(\mu;k) < m(k)$ *xos qiymatga ega bo’ladi va bu xos qiymatga mos xos funksiya quyidagi ko‘rinishda bo‘ladi :*

$$f_1(p) = \frac{\mu\, C}{\varepsilon_k(p) - \zeta_1} \in L_2^e(T^d), \quad C = const \neq 0\,. \tag{1}$$

b) d= 1,2 , Itiyoriy $\mu < 0$ *va* $k \in T^d$ *lar uchun* $h_\mu(k)$ *operator yagona* $M(k) < \zeta_1(\mu;k)$ *xos qiymatga ega bo‘ladi va bu xos qiymatga mos xos funksiya quyidagi ko‘rinishda bo‘ladi:*

$$f_1(p)=\frac{\mu C}{\varepsilon_k(p)-\zeta_1}\in L_e^2(\mathrm{T}^1),\quad C=const\neq 0. \qquad (2)$$

Teorema 1 ni isbotlash uchun quyidagi tasdiqlardan foydalaniladi.

Quyidagi to‘plamni kiritamiz $\Pi_0=\{\,k_1,\dots,k\in T^d:\ k_i\neq\ \pi\,,i=1\,,\dots,d\,\}$.

Tasdiq 1. *a) Ixtiyoriy* $k\in T^d=1{,}2$ *uchun* $a(k;\cdot)$ *funksiya* $\mathrm{C}\setminus[m(k),M(k)]$ *sohada analitik,* $(-\infty,m(k))$ *intervalda musbat va monoton o‘suvchi va mos ravishda ixtiyoriy* $k\in T^d$*,* $d=1{,}2$ *uchun* $a(k;\cdot)$ *funksiya* $\mathrm{C}\setminus[m(k),M(k)]$ *sohada analitik,* $(M(k),+\infty)$ *intervalda manfiy va monoton kamayuvchi; b) Ixtiyoriy* $k\in\Pi_0$ $d=1$ *uchun quyidagi tengliklar (asimptotik yoyilmalar) o‘rinli*

$$a(k;z)=\frac{(m(k)-z)^{-\frac{1}{2}}}{2\sqrt{\cos\frac{k}{2}}}+\frac{(m(k)-z)^{\frac{1}{2}}}{16\cos\frac{k}{2}\sqrt{\cos\frac{k}{2}}}+O(m(k)-z)^{\frac{3}{2}},\quad z\to m(k)-,$$

$$a(k;z)=\frac{(z-M(k))^{-\frac{1}{2}}}{2\sqrt{\cos\frac{k}{2}}}+\frac{(z-M(k))^{\frac{1}{2}}}{16\cos\frac{k}{2}\sqrt{\cos\frac{k}{2}}}+O(z-M(k))^{\frac{3}{2}},\quad z\to M(k)-,$$

$d=2$ bo‘lganda ham xuddi shunga o‘xshash yoyilma o‘rinli.

Tasdiq 2. *Ixtiyoriy* $\mu>0$ *(mos holda* $\mu<0$ *va* $k\in T^d$*,* $d=1{,}2$ *lar uchun shunday yagona* $\zeta_1(\mu;k)<m(k)$ *(mos holda* $M(k)<\zeta_1(\mu;k)$ *soni mavjudki, quyidagi tenglik o‘rinli bo‘ladi :*

$$\Delta(\mu,k;\zeta_1(\mu;k))=0 \qquad (mos\ holda\quad \Delta(\mu,k;\zeta_1(\mu;k))=0). \qquad (3)$$

Teorema 2. $d\geq 3$ $\mu>0$ *bo‘lsin . a)* $\mu>\mu_{min}^0(k)$ *va* $k\in T^d$ *bo‘lsin.*

U holda $h_\mu(k)$ *operator yagona* $\zeta_1(\mu;k)<m(k)$ *xos qiymatga ega bo‘ladi va bu xos qiymatga mos xos funksiya quyidagi ko‘rinishida bo‘ladi:*

$$f_1(p) = \frac{\mu\, C}{\varepsilon_k(p) - \zeta_1} \in L_2{}^e(T^d)\,, C = const \neq 0\,. \quad (4)$$

b) $\mu = \mu_{min}^0(k)$ *bo'lsin. U holda* $h_\mu(k)$ $operator$ $z = m(k)$ *nuqtada virtual sathga ega bo'ladi.*

c) $\mu < \mu_{min}^0(k)$ *bo'lsin. U holda* $h_\mu(k)$ $operator$ *muhim spektrdan tashqarida xos qiymatga ega bo'lmaydi.*

FOYDALANILGAN ADABIYOTLAR

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