ANIQ INTEGRALNING BA'ZI TATBIQLARI

A. Normatov

Qo'qon davlat pedagogika instituti.

Qo'qon sh. O'zbekiston

normatovadhamjon842@gmail.com

Annotatsiya: Maqolada aniq integralning ba'zi tatbiqlari to'g'risida so'z yuritilgan. Dastlab, shakllarning yuzini aniq integraldan foydalangan holda hisoblash formulasi keltirib chiqarilgan, so'ngra esa misol bilan mustahkamlangan. Keyin egri chiziqli trapetsiyadagi egri chiziq parametik usulda berilganda uning yuzini hisoblash formulasi topilgan va unga ham misollar keltirilgan. Misollar yechlshi bilan bir qatorda shakllar haqida ham qisqacha tarixiy ma'lumotlar berilgan.

Kalit so'zlar: to'g'ri chiziq, to'g'ri to'rtburchak, nomanfiy, uzluksiz, funksiya, ellips,sikloida, parametrik tenglama

Some applications of the definite integral

A. Normatov

Kokan State Pedagogical Institute.

Kokan. Uzbekistan

normatovadhamjon842@gmail.com

+998904051890

Abstract: The article discusses some applications of the definite integral. First, the formula for calculating the surface of shapes using the exact integral is derived, and then it is strengthened by an example. Then the formula for calculating the face of a curve in a curved trapezoid given by parametric method is found and examples are also given. In addition to the solution of the examples, brief historical information about the forms is given.

Key words: straight line, rectangle, non-negative, continuous, function, ellipse, cycloid, parametric equation

Некоторые приложения определенного интеграла

А. Норматов

Кокандский государственный педагогический институт.

Коканд. Узбекистан

normatovadhamjon842@gmail.com

Аннотация: В статье обсуждаются некоторые приложения определенного интеграла. Сначала выводится формула расчета поверхности фигур с помощью точного интеграла, а затем она усиливается на примере. Затем находится формула вычисления грани кривой в криволинейной трапеции, заданной параметрическим методом, и приводятся примеры. Помимо решения примеров даны краткие исторические сведения о фигурах.

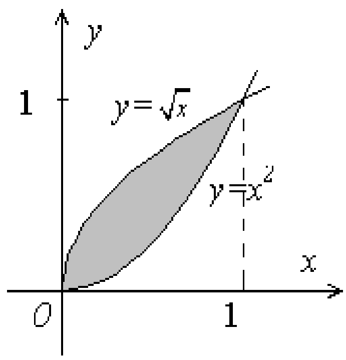
Ключевые слова: прямая, прямоугольник, неотрицательная, непрерывная, функция, эллипс, циклоида, параметрическое уравнение.

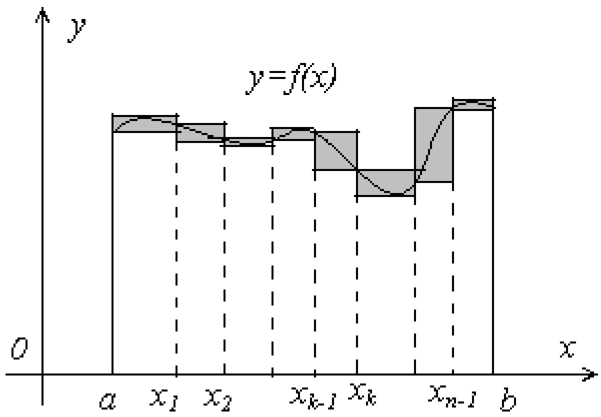
Yuzani hisoblash formulalalari. Faraz qilaylik, $x=a$, $x=b$, $y=0$ to‘g‘ri chiziqlar va $y=f(x)$ nomanfiy uzluksiz funksiya grafigi bilan chegaralangan D tekis figura berilgan bo‘lsin. Biz shu figuraning yuzini hisoblaymiz. Buning uchun $[a;b]$ kesmaning biror τ_n bo‘linishini olamiz:

$$a=x_0<x_1<...<x_n=b.$$

$f(x)$ ning $[x_{k-1},x_k]$ kesmadagi eng kichik va eng katta qiymatlari mos ravishda m_k va M_k bo‘lsin. Har bir $[x_{k-1},x_k]$ ga mos, asosi shu kesmadan iborat bo‘lgan, balandliklari esa $y=m_k$ va $y=M_k$ bo‘lgan ikkitadan to‘g‘ri to‘rtburchaklar yasaymiz.

Barcha to‘rtburchaklarning kichiklaridan (balandliklari m_k) iborat bo‘lgan ko‘pburchak D figuraga ichki chizilgan ko‘pburchak bo‘lib, katta to‘rtburchaklardan iborat ko‘pburchak tashqi chizilgan bo‘ladi. Ularning yuzlari mos ravishda





$$\sigma=\sum_{k=1}^{n}m_k\Delta x_k=\underline{S}(\tau_n),\quad \sigma'=\sum_{k=1}^{n}M_k\Delta x_k=\overline{S}(\tau_n)$$

bo‘ladi. Shartga ko‘ra $f(x)$ funksiya uzluksiz, bundan uning integrallanuvchi ekanligi kelib chiqadi. Demak,

$$\sup\sigma=\lim_{\lambda\to 0}\underline{S}(\tau_n)=\lim_{\lambda\to 0}\overline{S}(\tau_n)=\inf\sigma'\quad (\lambda=\max_{1\le k\le n}\Delta x_k)\,,$$

ya’ni D figura (egri chiziqli trapetsiya) kvadratlanuvchi va uning yuzi

$$S=\int_a^b f(x)dx$$

bo‘ladi. Agar yuqoridagi D figura quyidan $y=0$ to‘g‘ri chiziq o‘rniga $y=\varphi(x)$ $(\varphi(x)\le f(x),\quad x\in[a;b])$ chiziq bilan chegaralangan bo‘lib, $\varphi(x)$ funksiya uzluksiz bo‘lsa, u holda

$$S=\int_a^b (f(x)-\varphi(x))dx$$

bo‘ladi.

Misol. $y=x^2$ va $x=y^2$ chiziqlar bilan chegaralanagan figuraning yuzini toping.

Yechish. Berilgan figura yuqoridan $y=\sqrt{x}$, $0\le x\le 1$ chiziq bilan, quyidan esa $y=x^2$, $0\le x\le 1$ chiziq bilan chegaralangan. Shuning uchun

$$S=\int_0^1(\sqrt{x}-x^2)dx=\frac{2x^{\frac{3}{2}}}{3}\Bigg|_0^1-\frac{1}{3}x^3\Bigg|_0^1=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}.$$

Egri chiziqli trapetsiyadagi egri chiziq parametik usulda

$\begin{cases}x=\varphi(t),\\ y=\psi(t)\end{cases}$ $(\alpha\le t\le\beta)$ berilgan bo‘lsin, bunda $\varphi(\alpha)=a$, $\varphi(\beta)=b$, $[\alpha;\beta]$ kesmada $\psi(t)$ uzluksiz, $\varphi(t)$ esa monoton va uzluksiz $\varphi'(t)$ hosilaga ega deb faraz qilamiz. O‘zgaruvchini almashtirish qoidasiga asosan quyidagiga ega bo‘lamiz:

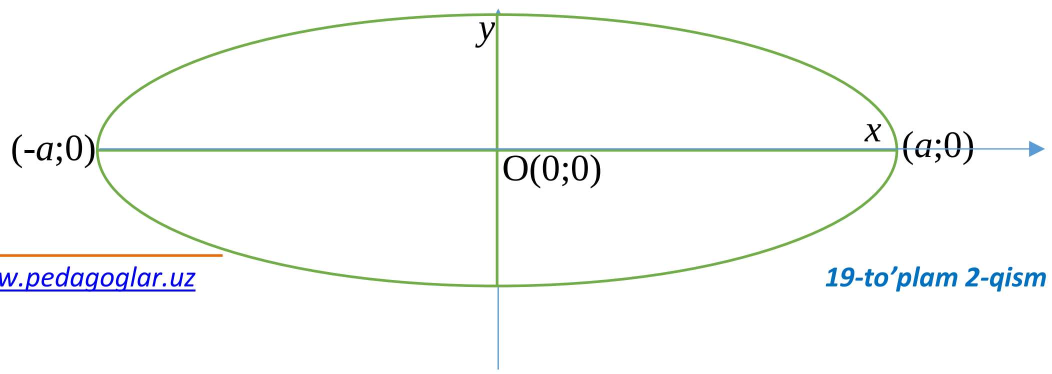
$$S=\int_a^b f(x)dx=\int_\alpha^\beta\psi(t)\varphi'(t)dt \qquad (1)$$

1-misol. $\begin{cases}x=a\cos t,\\ y=b\sin t\end{cases}$ ***$(0\le t\le 2\pi)$ ellipsning yuzini hisoblang***

(Ellips — ikkinchi tartibli yassi yopiq egri chiziq. Aylanma konus bilan uning uchidan o‘tmaydigan kesuvchi tekislik kesishishidan hosil bo‘ladigan yassi figura.

Ellipsning kanonik tenglamasi: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Ellipsning kichik yarim o‘qi b, katta yarim o‘qi a. Markazi esa O(0;0) – koordinata boshi. Ellipsning uchlari ($-a$;0), (a;0), (0;$-b$), (0;b). Ellipsning simmetriya markazi O(0;0), simmetriya o‘qlari Ox,

Oy o‘qlar)



(0;-*b*)

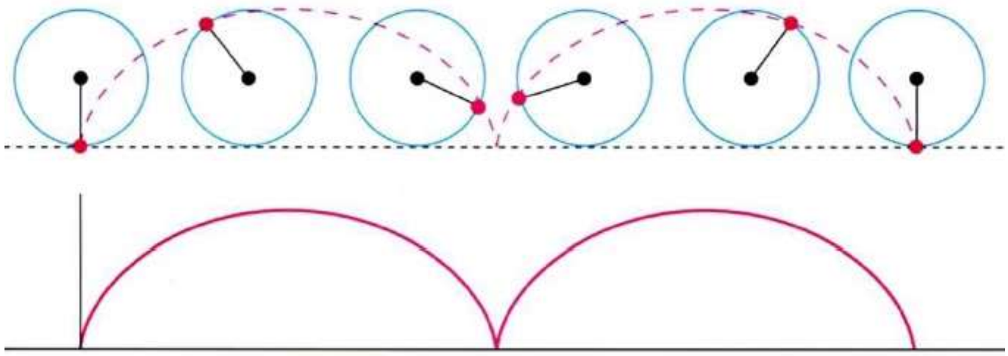
Yechish. ***Avval ellipsning chorak qismining yuzini topamiz:***

$$\frac{S}{4}=\int_{\frac{\pi}{2}}^{0} b\sin t(-a\sin t)dt = ab\int_{0}^{\frac{\pi}{2}}\sin^2 t dt = \frac{ab}{2}\int_{0}^{\frac{\pi}{2}}(1-\cos 2t)dt = \frac{ab}{2}\left(t-\frac{\sin 2t}{2}\right)\Bigg|_{0}^{\frac{\pi}{2}} = \frac{\pi ab}{4}$$

. Demak, $S=\pi ab$.

2-*misol*. *Ox* o‘qi va $\begin{cases} x=a(t-\sin t), \\ y=a(1-\cos t), \end{cases}$ $0\le t\le 2\pi$ sikloidaning bir arkasi bilan chegaralangan figura yuzini hisoblang.

(Sikloidalar to‘g‘ri chiziq bo‘ylab aylanma harakat qilayotgan aylanadagi aniq bir nuqtaning trayektoriyasi sifatida ta'riflanadi. Sikloidalarni ko‘plab mashhur matematiklar o‘rganishgan. Ushbu egri chiziqlarni mufassal tekshirgan dastlabki olimlardan biri mashhur olim, zamonaviy fizika fanining otasi bo‘lmish Galileo Galileydir (1564-1642). Biroq, Galileyning bu borada omadi chopgan deyish qiyin. Xususan u, sikloida va tekislik orqali chegaralangan yuzani hisoblashga ko‘p bora urinib, buni uddalay olmagan. U hatto metall plastinadan xuddi shunday egri chiziqli shaklni yasab, uning yuzasini sof fizik o‘lchashlar orqali ham hisoblamoqchi bo‘lgan, lekin baribir maqsadiga yeta olmagan. Ushbu shaklning yuzini aniq hisoblashni birinchi bo‘lib Rene Dekart (1598-1650) uddalagan.



U $3\pi r^2$ ga teng bo‘lib, bunda $\boldsymbol{r}$ - sikloida chizayotgan aylana radiusi. Dekartdan so‘ng Jil Roberval (1602-1675) ushbu egri chiziq chizayotgan yoy uzunligini hisoblab chiqdi. Ushbu yoy ham juda sodda matematik formula orqali ifodalanadi: L=8a. Ushbu egri chiziq bundan tashqari, ancha yillardan davomida ko‘plab olimlar va muhandislar uchun chaqilmas toshyong‘oq bo‘lib kelgan ikkita muhim masalani yechish uchun ham xizmat qildi)

Yechish. (1) formulaga ko‘ra

$$S=\int_0^{2\pi} a(1-\cos t)a(1-\cos t)dt = a^2\int_0^{2\pi}(1-\cos t)^2 dt =$$

$$= a^2(\int_0^{2\pi} dt - 2\int_0^{2\pi}\cos t dt + \int_0^{2\pi}\cos^2 t dt) = a^2((t-2\sin t)\Big|_0^{2\pi} +$$

$$+\frac{1}{2}\int_0^{2\pi}(1+\cos 2t)dt) = a^2(2\pi + \frac{1}{2}(t+\frac{1}{2}\sin 2t)\Big|_0^{2\pi}) = 3\pi a^2.$$

Adabiyotlar ro‘yhati

1. Normatov, A. (2022). SPATIAL OBJECTS. *Spectrum Journal of Innovation, Reforms and Development*, *4*, 586-590.

https://sjird.journalspark.org/index.php/sjird/article/view/180

2. Normatov, A. (2022). About the emergence of geometry. *Web of Scientist: International Scientific Research Journal*, *3*(7), 268-274.

https://wos.academiascience.org/index.php/wos/article/view/2200

3. Normatov, A. A., Tolipov, R. M., & Musayeva, S. H. Q. (2022). MAKTABLARDA MATEMATIKA FANINI O ‘QITISHNING DOLZARB

MASALALARI. *Oriental renaissance: Innovative, educational, natural and social sciences*, *2*(5), 1068-1075.

https://oriens.uz/journal/article/maktablarda-matematika-fanini-oqitishning-dolzarb- masalalari/

4. HH, M., AA, N., GB, U., & UA, M. (2022). COMPETENCE-BASED APPROACH IN THE PROFESSIONAL TRAINING OF FUTURE PRIMARY SCHOOL TEACHERS IN THE FIELD OF ICT. *International Journal of Early Childhood Special Education*, *14*(7).

https://www.int-jecse.net/abstract.php?id=5229

5. Normatov, A. (2022). APPLICATIONS OF THE DERIVATIVE. *Galaxy International Interdisciplinary Research Journal*, *10*(12), 1161-1164.

https://internationaljournals.co.in/index.php/giirj/article/view/3216

6. Normatov, A. (2022). Text problems. *INTERNATIONAL JOURNAL OF SOCIAL SCIENCE & INTERDISCIPLINARY RESEARCH ISSN: 2277-3630 Impact factor: 7.429*, *11*(11), 341-347.

https://www.gejournal.net/index.php/IJSSIR/article/view/1142

7. Gulira'no, A., & Adhamjon, N. (2022). Combinatory Elements in Primary Schools. *Journal of Pedagogical Inventions and Practices*, *4*, 32-33.

HTTPS://ZIENJOURNALS.COM

8. Норматов, А. А. (2023). ПОМОЩЬ УЧЕНИКАМ ПРИ РЕШЕНИИ НЕКОТОРЫХ ЗАДАЧ. *Conferencea*, 76-82.

https://conferencea.org

9. Shohista, M., & Normatov, A. (2022). MATEMATIK INDUKSIYA METODI. *Yosh Tadqiqotchi Jurnali*, *1*(5), 346-350.

https:// doi.org/ 10.5281/zenodo.6676876