NOFREDGOLM INTEGRAL TENGLAMALARINING AYRIM TADBIQLARI

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Annotasiya: Ushbu maqolaning mazmuni Nofredgolm integral tenglamalarining ushbu ikkita ya'ni Integral tenglama yadrosi argumentlar ayirmasidan iborat bo'lgan hol va Integral tenglama yadrosi chizig’ida birinchi tartibli maxsuslikka ega bo’lgan hollarda ko'rib chiqish va shu orqali kerakli natijani hosil qilish

Kalit so’zlar: Nofredgolm integral tenglamasi, Furye almashtirishi, uzoqlashuvchi, yopiq kontur, Soxotskiy-Plemel formulasi, regulyar funksiya

1-HOL: Integral tenglama yadrosi argumentlar ayirmasidan iborat bo’lgan hol.

$$\varphi(x)-\lambda\int_{-\infty}^{+\infty}e^{-|x-s|}\varphi(s)ds=f(x) \qquad (1)$$

integral tenglamani o’rganamiz.

(1) integral tenglama Fredgolm tenglamasi bo’lishi uchun

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|K(x,s)|^2dxds<+\infty \qquad (2)$$

bo’lishi zarur edi.

(1) tenglama uchun (2) shartni buzulganligini ko’rsatamiz ya’ni $K(x,s)=e^{-|x-s|}$ yadro kvadrati bilan integrallanuvchi emas. Haqiqatdan ham

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|K(x,s)|^2dxds=\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}e^{-2|x-s|}ds \qquad (3)$$

ichki integralni hisoblaymiz

$$\int_{-\infty}^{\infty}e^{-2|x-s|}ds=\int_{-\infty}^{x}e^{-2(x-s)}ds+\int_{x}^{+\infty}e^{-2(x-s)}ds=\frac{1}{2}e^{-2(x-s)}\Big|_{-\infty}^{x}-\frac{1}{2}e^{-2(x-s)}\Big|_{x}^{+\infty}=\frac{1}{2}-0-0+\frac{1}{2}=1$$

Bu tenglikdan

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|K^2(x,s)|^2dxds=\infty \qquad (4)$$

ekanligi kelib chiqadi. Demak (1) tenglama Fredgolm tenglamasi emas. Endi (1) tenglamani echish usulini keltiramiz. Tenglamaning ikki tomoniga Furye almashtirishni qo'llaymiz buning uchun ikki tomonini $\frac{1}{\sqrt{2\pi}}e^{-ixt}$ ko'paytirib $(-\infty,+\infty)$ oraliqda integrallaymiz

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ixt}\varphi(x)dx-\frac{\lambda}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-ixt-|x-s|}\varphi(s)ds\,dx=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ixt}f(x)dx \qquad (5)$$

va ushbu

$$\Phi(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ixt}\varphi(x)dx$$

$$F(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ixt}f(x)dx$$

belgilashlarni kiritib ushbu

$$I=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-ixt-|x-s|}\varphi(s)ds\,dx=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\varphi(s)ds\int_{-\infty}^{+\infty} e^{-ixt-|x-s|}dx \qquad (6)$$

integralni hisoblaymiz.

Ichki integralda ushbu $y=x-s$ almashtirishni bajaramiz, u holda

$$\int_{-\infty}^{\infty} e^{-ixt-|x-s|}dx=e^{-ist}\int_{-\infty}^{\infty} e^{-iyt-|y|}dy=e^{-ist}\left(\int_{-\infty}^{0} e^{y(1+it)}dy+\int_{0}^{\infty} e^{-y(1+it)}dy\right)=e^{-ist}\left(\frac{1}{1-it}e^{y(1-it)}\Big|_{-\infty}^{0}-\frac{1}{1+it}e^{y(1-it)}\Big|_{-\infty}^{0}\right)=$$

$$=e^{-ist}\left(\frac{1}{1-it}+\frac{1}{1+it}\right)=\frac{2e^{-ist}}{1+t^2} \qquad (7)$$

shunday qilib (7) ga asosan (6) ifodani quyidagicha yozib olamiz

$$I=\frac{1}{\sqrt{2\pi}(1+t^2)}\int_{-\infty}^{+\infty} e^{-ixt}\varphi(s)ds=\frac{1}{1+t^2}\Phi(t) \qquad (8)$$

(8) ga asoasn (5) ifoda sodda tenglamaga keladi

$$\left(1-\frac{2\lambda}{1+t^2}\right)\Phi(t)=F(t)$$

agar $\lambda<\frac{1}{2}$ bo'lsa, u holda $1-\frac{2\lambda}{1+t^2}\neq 0$. Demak

$$\Phi(t)=\frac{1+t^2}{1+t^2-2\lambda}F(t)$$

yechimga kelamiz. Bu erdan

$$\varphi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ixt}\Phi(t)dt=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ixt}\frac{1+t^2}{1+t^2-2\lambda}F(t)dt \qquad (9)$$

Yechimga kelamiz.

Agar $\lambda \geq \frac{1}{2}$ bo’lsa umuman olganda (9) integral mavjud emas, va (1) tenglama yechimga ega emas (λ-xarakteriskik son mazmuni) lekin bu sonlar butun integralni to’ldiradi, bu Fredgolm teoremasiga zid. Fredgolm tenglamasida λ-xarakteriskik son yakkalangandir.

Shunday o’xshash natijani

$$\varphi(x) - \lambda \int_{-\infty}^{\infty} K(x-s)\varphi(s)ds = f(x)$$

tenglama uchun ham olish mumkin faqat tenglama yadrosi uchun

$$\int_{-\infty}^{\infty} |K(x)| dx < \infty$$

shartni bajarilishi kerak.

2-HOL: Integral tenglama yadrosi $x = t$ chizig’ida birinchi tartibli maxsuslikka ega bo’lgan hol.

$\Gamma - z = x + iy$ kompleks tekislikda yotuvchi yopiq kontur bo’lsin

$$\int_{\Gamma} \frac{\varphi(\xi)}{\xi - t} d\xi, \quad \xi \in \Gamma \qquad (10)$$

(10) integral oddiy ma’noda uzoqlashuvchi bo’ladi, lekin bu integral integralning bosh qiymati ma’nosida yaqinlashuvchi bo’ladi. t nuqtani Γ chiziqdan markazi t nuqtada radiusi ε bo’lgan aylana bilan ajratib olamiz va qolgan qismini Γ_{ε} orqali belgilaymiz. Singular integral ushbu

$$\int_{\Gamma} \frac{\varphi(\xi)}{\xi - t} d\xi = \lim_{\varepsilon \to 0} \int_{\Gamma_{\varepsilon}} \frac{\varphi(\xi)}{\xi - t} d\xi$$

tenglik bilan aniqlanadi.

$z - \Gamma$ chiziqda yotmaydigan ixtiyoriy nuqta bo’lsin.

$$\Phi(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{\varphi(\xi)}{\xi - z} d\xi$$

funksiyani kiritamiz

$z \to t \in \Gamma$ ga integrallansin $\Phi_t(t)$ va $\Phi_e(t)$ orqali $\Phi(z)$ ning $z \to t$ ga mos ravishda Γ kontur ichidan yoki tuchqarisida integragandagi limit qiymatin belgilaymiz.

U holda Soxotskiy-Plemel formulasiga ko’ra

$$\Phi_i(t) = \frac{1}{2}\varphi(t) + \frac{1}{2\pi i} \int_{\Gamma} \frac{\varphi(\xi)}{\xi - t} d\xi \qquad (11)$$

$$\Phi_e(t) = -\frac{1}{2}\varphi(t) + \frac{1}{2\pi i} \int_{\Gamma} \frac{\varphi(\xi)}{\xi - t} d\xi \qquad (12)$$

tengliklar o’rinlidir.

$\varphi(z)$ funksiya Γ kontur ichida regulyar funksiya bo'lsin va Γ ga qadar uzluksiz bo'lsin. Agar z Γ kontur tashqarisda bo'lmasa.

$\Phi(z)\equiv 0$ va $\Phi_e(t)\equiv 0$ yoki Soxotskiy-Plemel formulasiga ko'ra

$$\varphi(t)-\frac{1}{\pi i}\int_{\Gamma}\frac{\varphi(\xi)}{\xi-t}d\xi\equiv 0 \qquad (13)$$

Bu tenglik $\lambda=1$ sonu

$$\varphi(t)-\frac{\lambda}{\pi i}\int_{\Gamma}\frac{\varphi(\xi)}{\xi-t}d\xi\equiv 0 \qquad (14)$$

singulyar integral tenglamaning echimi ekanligini bildiradi va bu xarakteristik songa Γ kontur ichida Γ chiziqqacha regulyar funksiyaning Γ chiziqdagi qiymati xos funksiyaga mos keladi.

Endi $\varphi(z)$ Γ chiziqdan tashqari sohada regulyar bo'lib chiziqgacha uzluksiz bo'lsin va $\varphi(\infty)=0$.

U holda Γ chiziq ichda $\Phi(z)\equiv 0$ va $\Phi_i(t)\equiv 0$ bu tenglikdan Soxotskiy-Plemel formulasiga ko'ra

$$\varphi(t)+\frac{1}{\pi i}\int_{\Gamma}\frac{\varphi(\xi)}{\xi-t}d\xi\equiv 0 \qquad (15)$$

tenglikka kelamiz.

Bundan (15) tenglama yani bir $\lambda=-1$ xarakteristik songa ekanligi kelib chiqadi va bu songa cheksiz ko'p xos funksiyalar mos keladi. Shunday qilib Fredgolmning har qanday xarakteristik songa chekli sondagi xos funksiyalar mos kelishi haqidagi teorema buzuladi.

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